Quintic 4 Education

Q4E Case Studies

Topic 3 – Coefficient of Restitution

Proposed subject usage:

Mathematics (A Level)
Sports Science ($1^{st}/2^{nd}$ yr)

e.g. AQA A-level Physics 2005 Specification

Newton's law of restitution. Applications to direct impact of two particles and normal impact of a particle with a plane surface.

Knowledge of the terms perfectly elastic (1 = *e*) and perfectly inelastic/plastic (0 = *e*) is expected.

Coefficient of Restitution

Direct impacts of two objects occur when their velocity vectors are parallel. Linear momentum is conserved but energy is lost during the impact when one or both of the objects are deformed. The greater the deformation that occurs the longer the impact lasts. The strain energy stored during the deformation will partly be regained in the latter part of the impact depending on how plastic or elastic the impact is. Newton's empirical law demonstrates this:

$$u_1 - u_2 = e(v_1 - v_2)$$

Where: u represents the approach velocity
v is the separation velocity
1 and *2* represent the two objects
e = a constant known as the coefficient of restitution

Method 1- Using velocities to determine e

If the impact is between an object (e.g. a tennis ball) and a surface then there is no velocity prior to or after impact for object 2.

Therefore: as $u_2 = v_2 = 0$ in the above equation

$$v/u = e$$

Method 2 - Using heights to determine *e*

$v^2 = u^2 + 2as$ *(*one equation of uniform motion*)*

Where: a = gravity (g)
s = distance (in this case height)

Therefore: $u = \sqrt{2gh_d}$
$v = \sqrt{2gh_b}$

Where: h_d = height dropped (prior to impact)
h_b = height bounced (after impact)

By substituting $u = \sqrt{2gh_d}$ and $v = \sqrt{2gh_b}$ *into* $v/u = e$

$$e = \sqrt{(h_b / h_d)}$$

This law indicates how *e* and velocity before impact affects movement after impact. The value of *e* can range from 0 – 1 with 0 being a perfectly plastic impact and 1 being a perfectly elastic impact.

Methods

Five different sports balls dropped onto a wooden table and videoed at high speed. The videos were digitised and the data were exported into excel files. Heights of the balls were determined using the distance measuring tool within Quintic Biomechanics 9.03 (Quintic Lite, Sports and Coaching users can calculate distance by using the lines function and on-screen coordinates). Both methods of calculating *e* (using bounce height and velocity) can be used and compared.

Functions of the Quintic Software used:

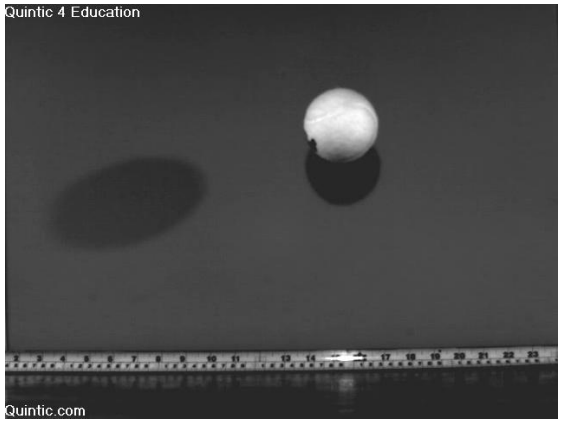
- Digitisation module
- Calibration
- Export data
- Lines and distance measuring tool
- Split screen
- On-screen point coordinates

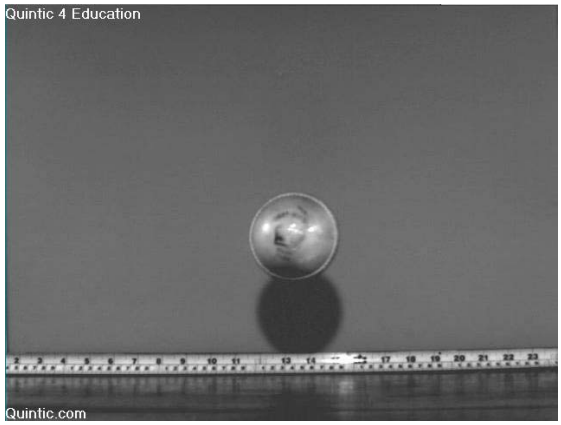
Results

The results shown in the tables below indicate that there is a distinct difference between the impacts of the different balls onto the surface.

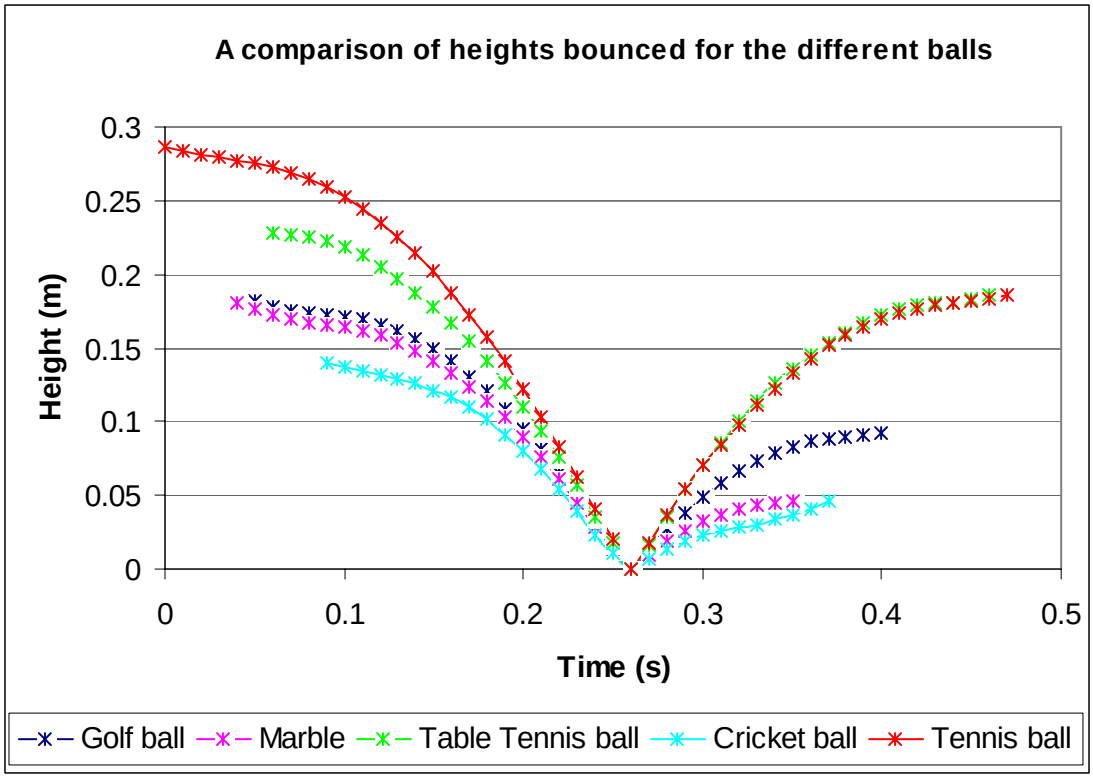
Method 1	**Table Tennis Ball**	**Marble**	**Cricket Ball**	**Tennis Ball**	**Golf Ball**
Velocity prior to impact (*u*) (m/s)	1.92	1.55	1.39	2.09	1.66
Velocity after impact (*v*) (m/s)	1.8	0.82	0.6	1.76	1.23
e	**0.94**	**0.53**	**0.43**	**0.84**	**0.74**

Method 2	Table Tennis Ball	Marble	Cricket Ball	Tennis Ball	Golf Ball
H dropped (m)	0.22	0.16	0.13	0.27	0.16
H bounced (m)	0.18	0.05	0.03	0.18	0.1
Hb/Hd	0.82	0.31	0.23	0.67	0.63
e	**0.90**	**0.56**	**0.48**	**0.82**	**0.79**





The heights bounced can be compared across the different balls in the following graph:



The coefficients of restitution vary according to the ball used; the tennis ball had the highest value (e = 0.82) and the cricket ball the lowest value (e = 0.48). This means the tennis ball bounced much higher than the cricket ball because much less energy was lost during impact. Therefore the tennis ball did not decelerate as much and rebounded with a higher velocity.

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Conclusion

Coaches can apply this study advantageously to increase knowledge of their sport by comparing different sports, balls and surfaces. Understanding of biomechanical factors involved in sport will be beneficial to the athlete and can be used to predict and enhance performance. Quintic Software is ideal for capturing and analysing video footage of athletes and applying biomechanical knowledge to training and coaching strategies of both individuals and the club or team.

Teachers may use the Quintic Software to engage students' interest by applying the fundamental principles of topics such as the coefficient of restitution to sporting situations, using real-life video examples. For example, from this case study it is evident that the coefficient of restitution is very important in sport, as different balls will behave differently in similar situations. Perhaps it would be interesting to question the implications of using certain sports balls in other sports or discuss the benefits of warming up a squash ball. Using Quintic Software teachers can make the syllabus more interesting and enable increased understanding of difficult topics by putting them in an appealing sporting context.

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